Problems on Binomial Distribution :-

→ Formula used in Binomial Distribution is

$$P(r) = {}^{n}C_{r} \cdot p^{r} \cdot q^{(n-r)}$$

where, $P(r)$ = Probability of 'r' success.

n = Number of trials

p = Probability of success

r = Number of successes in 'n' trails.

(1) QN-1. A coin is tossed 6 times. What is the probability of getting a) 3 heads, b) At most 2 heads c) At least 4 heads.

Given, $n=6$, $p=0.5$, $q=0.5$ (Assumption $p=50\%$ & $q=50\%$) success & failure.

a) 3 heads :-

$r=3$.

$$P(r) = {}^{n}C_{r} \cdot p^{r} \cdot q^{(n-r)}$$

$$P(3) = {}^{6}C_{3} \cdot (0.5)^3 (0.5)^{(6-3)}$$

$$= {}^{6}C_{3} (0.5)^3 (0.5)^3$$

$$= 20 (0.125)(0.125)$$

$$= 0.3125$$

$${}^{6}C_{3} = \frac{6!}{(6-3)!\,3!} = \frac{6\times5\times4\times\cancel{3!}}{3\times2\times1\times\cancel{3!}} = \frac{120}{6} = 20$$

b) At most 2 heads :-

P(At most 2 heads) = P(0 Head) + P(One Head) + P(Two Head)

$\therefore r=0,\ r=1,\ r=2$

when $r=0$

$$P(0) = {}^{6}C_{0} (0.5)^0 (0.5)^{(6-0)}$$

$$= 1 \times 1 \times 0.015625$$

$$= 0.015625$$

when $r=1$

$P(1) = {}^6C_1 (0.5)^1 (0.5)^{(6-1)}$

$= 6 \times 0.5 \times 0.03125$

$= 0.09375.$

when $r=2$

$P(2) = {}^6C_2 (0.5)^2 (0.5)^{(6-2)}$

$= 15\,(0.25)(0.5)^4$

$= 15\,(0.25)(0.0625)$

$= 0.234375.$

P(At most 2 heads) $= 0.015625 + 0.09375 + 0.234375$

$\therefore$ P($\leq$ 2 heads) $= \underline{0.34375}$

c) At least 4 heads:-

P(At least 4 heads) = P(4 heads) + P(5 heads) + P(6 heads)

$r=4,\ r=5,\ \&\ r=6$

when, $r=4$

$P(4) = {}^6C_4 (0.5)^4 (0.5)^{(6-4)}$

$= {}^6C_4 (0.5)^4 (0.5)^2$

$= 15 \times 0.0625 \times 0.25$

$= \underline{0.234375}$

${}^6C_4 = \dfrac{6!}{(6-4)!\,4!}$

$= \dfrac{6 \times 5 \times \cancel{4!}}{2 \times 1 \times \cancel{4!}}$

$= \dfrac{30}{2} = \underline{15}$

when, $r=5$

$P(5) = {}^6C_5 (0.5)^5 (0.5)^{(6-5)}$

$= {}^6C_5 (0.5)^5 (0.5)^1$

$= \cancel{6 \times 0.01562 \times 0}.5$

$= 6 \times 0.03125 \times 0.5$

$= \underline{0.9375}$

when, $r=6$

$P(6) = {}^6C_6 (0.5)^6 (0.5)^{(6-6)}$

$= 1\,(0.5)^6 (0.5)^0$

$= 1 \times 0.015625 \times 1$

$= \underline{0.015625}$

P(At least 4 heads) $= 0.234375 + 0.09375 + 0.015625$

P(At least 4 heads) $= \underline{0.34375}$

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(2)

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$n = 5, \quad p = 0.3, \quad q = 1 - p = 1 - 0.3 = 0.7$

a) 4 games:

$r = 4$

$P(4) = {}^5C_4 (0.3)^4 (0.7)^{5-4}$

$= {}^5C_4 (0.3)^4 (0.7)^1$

$= 5 (0.0081)(0.7)$

$= 0.02835$

${}^5C_4 = \frac{5!}{(5-4)!4!} = \frac{5!}{1!4!} = \frac{5 \times 4!}{1 \times 4!} = 5$

b) At least 2 games:

P (At least 2 games) = P(2 games) + P(3 games) + P(4 g) + P(5 g)

at $r = 2$, $r = 3$, $r = 4$, & $r = 5$,

when, $r = 2$:

$P(2) = {}^5C_2 \cdot (0.3)^2 (0.7)^{5-2}$

$= 10 (0.3)^2 (0.7)^3$

$= 10 (0.09)(0.343)$

$= 0.3087$

when $r = 3$

$P(3) = {}^5C_3 (0.3)^3 (0.7)^{(5-3)}$

$= 10 (0.027)(0.49)$

$= 0.1323$

when, $r = 4$:

$P(4) = {}^5C_4 (0.3)^4 (0.7)^{5-4}$

$= 5 (0.3)^4 (0.7)^1$

$= 5 (0.0081)(0.7)$

$= 0.02835$

when $r = 5$

$P(5) = {}^5C_5 (0.3)^5 (0.7)^{5-5}$

$= 1 (0.00243)(0.7)^0$

$= 1 (0.00243) 1$

$= 0.00243$

P (At least 2 games) = $P(r=2) + P(r=3) + P(r=4) + P(r=5)$

$= 0.3087 + 0.1323 + 0.02835 + 0.00243$

$= 0.47178$

3. A box contains 200 ~~orag~~ oranges, out of which 40 are defective, 10 oranges are selected for inspection. Find out the probability that
(a) all 10 are good (b) all 10 are defective
(c) atleast 2 is defective (d) at the most 3 are defective

Given:-

$n = 10$, $P = \dfrac{\text{defective oranges}}{\text{Total no. of oranges}} = \dfrac{40}{200} = 20\% = \underline{0.2}$

$q = 1 - P = 1 - 0.2 = \underline{0.8}$

(a) All 10 are good:- (means no defective oranges)

$r = 0$, $\cancel{{}^{10}C_0 = (0.2)^0}$,

$P(0) = {}^{10}C_0 (0.2)^0 . (0.8)^{10-0}$
$= {}^{10}C_0 (0.2)^0 (0.8)^{10}$
$= 1 \times 1 \times 0.1073$
$= \underline{0.1073}$

(b) All 10 are defective

$r = 10$, $P(10) = {}^{10}C_{10} (0.2)^{10} (0.8)^{10-10}$
$= 1 (0.2)^{10} (0.8)^0$
$= 1 \times 0.000001 \times 1$
$= \underline{0.000001}$

(c) Atleast 2 is defective:- (minimum 2 is defective)

$P(P \geq 2) = 1 - P(0) + P(1)$

when $r = 0$

$P(0) = {}^{10}C_0 (0.2)^0 . (0.8)^{10}$
$= 1 \times 1 \times 0.1073$
$= \underline{0.1073}$

when $r = 1$

$P(0) = {}^{10}C_1 (0.2)^1 . (0.8)^{10-1}$
$= 10 (0.2) (0.1342)$
$= \underline{0.2684}$

Continued

$(P \geq 2) = 1 - P(0) + P(1)$

$= 1 - (0.1073 + 0.2684)$

$= 1 - 0.3757$

$= 0.6243$

(d) At the most 3 are defective – (maximum 3)

$(P \geq 3) = P(0) + P(1) + P(2) + P(3)$

when $r = 0$

$P(0) = {}^{10}C_0 (0.2)^0 (0.8)^{10-0}$

$= 1 \times 1 \times 0.1073$

$= 0.1073$

when, $r = 1$

$P(1) = {}^{10}C_1 (0.2)^1 (0.8)^{10-1}$

$= 10 (0.2)(0.1342)$

$= 0.2684$

when $r = 2$

$P(2) = {}^{10}C_2 (0.2)^2 (0.8)^{10-2}$

$= 45 (0.04)(0.8)^8$

$= 45 (0.04)(0.1676)$

$= 0.30168$

when $r = 3$

$P(2) = {}^{10}C_3 (0.2)^3 (0.8)^{(10-3)}$

$= 120 (0.008)(0.2095)$

$= 0.20112$

$P(\text{At most } 3) = 0.1073 + 0.2684 + 0.30168 + 0.20112$

$= 0.8785$